Mathematics I

RATIO AND PROPORTION

- **Ratio** gives us a relation between two quantities having similar unit. The ratio of A to B is written as A : B or $\frac{A}{B}$.

- **Proportion** is an expression in which two ratios are equal. For example

  $\frac{A}{B} = \frac{C}{D}$ can be written as

  A : B : : C : D

- a : b = ma : mb

- a : b : c = A : B : C can be written as

  $$\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$$

- If a : b = c : d, then

  $$\frac{a}{b} = \frac{c}{d}$$

  $$\frac{b}{a} = \frac{d}{c}$$

  $$\frac{a}{c} = \frac{b}{d}$$

- If a number is divided into three parts, whose ratio is a : b : c then

  First part = $\dfrac{a}{a+b+c} xZ$

  Second part = $\dfrac{b}{a+b+c} xZ$

  Third part = $\dfrac{c}{a+b+c} xZ$

- If a : b = c : d, then

  $$\frac{(a+b)}{b} = \frac{(c+d)}{d}$$

  and $\dfrac{(a-b)}{b} = \dfrac{(c-d)}{d}$

- If a : b = c : d, then

  Product of means = product of extremes

  ∴ b x c = a x d

- **Continued Proportion**: If three numbers a, b and c are in continued proportion, then $b^2 = ac$ i.e. $b = \sqrt{ac}$

  Here we can say that a is called *first proportion*, c is called *third proportion* and b is called *mean proportion*.

- If two nos. are given, and you are required to find *mean proportion*, then it should be written as a : x :: x : b, and is *third proportion* is to be computed, then it should be written as a : b :: : x.

- If in a partnership the investments made by first, second and third partners are x_1, x_2, x_3 respectively, the time period be t_1, t_2, t_3 then the ratio of profits is given by $x_1 t1 : x_2 t_3 : x_3 t_3$.

- If $x_1 : x_2 : x_3$ is the ratio of investments and $P_1 : P_2 : P_3$ be the ratio of Profit then time periods are given then

  $$\frac{P_1}{x_1} : \frac{P_2}{x_2} : \frac{P_3}{x_3}$$

- If $P_1 : P_2 : P_3$ is the ratio of profit and $t_1 : t_2 : Pt_3$ be the ratio of time periods then ratio of investments are given then

  $$\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3}$$

- **Mixtures and allegations** is about mixing different objects in order to get desired levels/percentage/concentration of different objects.

  When two ingredients A and B of quantities q_1 and q_2 and cost price/unit x_1 and x_2, are mixed to get a mixture x having cost price/unit x_m, then

$$\frac{q_1}{q_2} = \frac{x_2 - x_m}{x_m - x_1}$$

and $x_m = \dfrac{x_1 q_1 + x_2 q_2}{q_1 + q_2}$

- If a mixture contains two ingredients A and B in the ratio x : y, then percentage of A in the mixture

$$= \frac{x}{x+y} x 100\%$$

and % of B will be

$$= \frac{y}{x+y} x 100\%$$

Example

A grocer mixes coffee powder of 2 types, one of which is priced at Rs. 60 and the other at Rs. 90. What should be the ratio of combining the two, to sell the blended mix coffee powder of the two types at Rs. 80?

(a) 2:1 (b) 2:3

(c) 1:2 (d) 3:2

Answer

(c)

Example

In a 120 litre of solution of Acid and water, acid is 75%. A person takes out 20 litres of this solution and added 16.2 litres of acid and 3.8 litres of water in the remaining solution. What is the percentage of water in the final solution?

(a) 22 (b) 24

(c) 25 (d) 28

Answer

(b) In 100 l, Acid = 75 l and water = 25 l.

when 16.2 l acid and 3.8 l water is added the water = 25 + 3.8 = 28.8 l. Now, total

volume of solution = 100 + 16.2 + 3.8 = 120 l

∴ Water % = (28.8/120) x 100 = 24%

Example

In a University, for each ₹ 200 spent by the Cultural Committee, Debating Committee spends ₹ 20 and for every ₹ 400 spent by the Debating Committee, the Student Welfare Committee spends ₹ 150. The triple ratio of the money spent by the Cultural Committee to the money spent by the Debating Committee to the money spent by the Student Welfare Committee can be expressed as

(a) 80 : 8 : 3 (b) 60 : 8 : 3

(c) 40 : 4 : 3 (d) 20 : 4 : 3

Answer

(a) CC : DC : WC

200 : 20 → : 20

400 : ← 400 : 150

_____

80000 : 8000 : 3000

80 : 8 : 3

Example

1. A vessel contains a mixture of milk and water in the ratio of 5:3 respectively. How much of the mixture must be siphoned off and replaced with water, so that the mixture may be half milk and half water?

(a) 1/7 (b) 1/4

(c) 1/5 (d) 1/3

2. Praveen has ₹ 4,662 in the form of 2, 5 and 10 rupee notes. If these notes are in the ratio of 3:5:8, the number of five rupees notes with him is:

(a) 336 (b) 250

(c) 84 (d) 210

3. Gold and copper are as heavy as water by 19 and 9 times respectively. The ratio in which these two metals be mixed so that the mixture is 17 times as heavy as water is:

(a) 2:3   (b) 3:4

(c) 3:2   (d) 4:1

Answers

1. (c) Let the vessel contains 8 litres of liquid. Let x litres of this liquid be replaced with water. Now quantity of water in new mixture will be

$\left(3 - \dfrac{3x}{8} + x\right)$ litres and quantity of milk in new mixture will be $\left(5 - \dfrac{5x}{8}\right)$ litres.

$\therefore \left(3 - \dfrac{3x}{8} + x\right) = \left(5 - \dfrac{5x}{8}\right)$ which

gives x = 8/5. So, part of mixture replaced

$= \left(\dfrac{8}{5} x \dfrac{1}{8}\right) = \dfrac{1}{5}$

2. (d) Let the number of 2,5 and 10 rupees notes be 3x, 5x and 8x. Then, sum of their values = 3x × 2 + 5x × 5 + 8x × 10 = 111 x. Hence x = 42.

3. (d) (19 g + 9 c) / (g + c) = 17

19 g + 9 c = 17 g + 17 c

2 g = 8 c Hence g/c = 4/1

Example

60 kg of an alloy X is mixed with 100 kg of an alloy Y. If alloy X has lead and tin in the ratio of 3:2 and alloy Y has tin and copper in the ratio of 1:4, then the amount of tin in the new alloy is.

(a) 53 kgs.   (b) 80 kgs.

(c) 44 kgs.   (d) 36 kgs.

Answer

(c) Total amount of tin in alloy x and y = (2x/5) + (y/5) = (2/5) x 60 + (100/5) = 44 Kg.

Example

Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. What is the weight of dry grapes available from 20 kg of fresh grapes?

(a) 2 kg   (b) 2.4 kg

(c) 2.5 kg   (d) 2.6 kg

Answer

(c) Weight of *non water* in 20 kg of fresh grapes = (10/100) x 20 = 2 kg. Let dry fruit is x kg. which contains 80% non water. Hence, we can say 2 = (80/100) x (x) i.e. x = 2.5 kg.

Example

1. The ratio in which Aman and Bimal have contributed to the capital of a company is 3 : 4. Bimal has invested his capital for only 3 months and has received half as much profit as Aman, at the end of the year. Find out for how much time has Aman invested his capital in the company.

(a) 8 months   (b) 14 months

(c) 15 months   (d) 1 year

2. Two whole numbers whose sum is 64 can be in the ratio of ?

(a) 7 : 2   (b) 7 : 6

(c) 3 : 1   (d) 8 : 7

Answers

1. (a) Aman and Bimal invested in ratio = 3 : 4 Bimal invested for 3 month where as Aman for T month effective investment Ratio = 3t : 4 × 3 = 3t : 12 = t : 4. Hence t = 8 months.

2. (c) From option 3 : 1, 3 + 1 = 4 is only factor of 64.

Example

1. A person spends 1/3 part of his income on food, 1/4 part on house rent and remaining ₹ 630 on other items. The house rent is

   (a) ₹ 504   (b) ₹ 1512

   (c) ₹ 378   (d) None of these

2. An employer reduces the number of employees in the ratio 8 : 5 and increases their wages in the ratio 7 : 9. Therefore, the overall wages bill is

   (a) increased in the ratio 45 : 56

   (b) decreased in the ratio 56 : 45

   (c) increased in the ratio 13:17

   (d) decreased in the ratio 72 : 35

3. A starts a business with ₹ 5000 and B joins the business 5 months later with an investment of ₹ 6000. After a year, they earn a profit of ₹ 34000. Find the shares of A and B in the profit amount depending on their individual investment.

   (a) ₹ 20000, ₹ 14000

   (b) ₹ 16000, ₹ 16000

   (c) ₹ 14000, ₹ 20000

   (d) None of these

Answer

1. (c) Let the money is x. Now as per question x - (x/3) - (x/4) = 630 ∴ x = ₹ 1512.

2. (b) Let no. of employee be = 8x and 5x

   Increase in wages be = 7y, 9y

   Wage bill earlier = 8x × 7y = 56xy

   Wage bill afterward = 5x × 9y = 45xy

   56xy/45xy = 56 : 45

3. (a) Effective price contributed by A = 5000 × 12 = 60000

   Effective price contributed by B = 6000 × 7 = 42000

Ratio = 60,000/42,000 = 10/7

Total profit = 34,000

A gets = 34000 x (10/17) = ₹ 20,000

B gets = 34000 x (7/17) = ₹ 14,000

Example

Angles of a quadrilateral are in the ratio 3 : 4 : 5 : 8. The smallest angle is

(a) 20° (b) 54°

(c) 36° (d) 18°

Answer

(b) Let the angles be 3x, 4x, 5x, and 8x

3x + 4x + 5x + 8x = 360° ∴ x = 18°

∴ Smallest angle = 3 × 18° = 54°

Example

18. A sum was divided among P, Q & R. R got double than P who got double than Q. If the difference between the shares of Q and R is ₹ 3675.00, then the sum in rupees is

    (a) 4900   (b) 8575

    (c) 11025   (d) 7350

19. If the ratio of the areas of two squares is 25 : 36, then the ratio of their perimeters is

    (a) 5 : 6   (b) 25 : 36

    (c) 6 : 5   (d) 36 : 25

Answers

1. (b) R = 2P and P = 2 Q ∴ R = 2P = 4Q

   ∴ R/4 = P/2 = Q/1

   Ratio in which sum is divided among R, P and Q = 4 : 2 : 1.

Let sum be 4x, 2x and   x.

$\therefore$ 4x - x = 3675 $\therefore$ x = 1225.

Total sum = 7x = 7 x 1225 = 8575.

2.   (a) Given ratio = 25 : 36. Ratio of sides will be R$\sqrt{25}/\sqrt{36}$ = 5/6. Hence ratio of parameter is (4 x 5)/(4 x 6) = 5/6

Examples

1.   Raju earns twice as much in March as in each of the other months of the year. What part of his annual earnings he earns in that month?

   (a) 1/5   (b) 5/7

   (c) 2/13   (d) 1/10

2.   The ratio of milk and water in 60 Litres of adulterated milk is 2: 1. If the ratio of milk and water is to be 1 : 2, then the amount of water to be added further is

   (a) 20 Litres   (b) 30 Litres

   (c) 40 Litres   (d) 60 Litres

3.   The monthly salary of A, B and C are in the ratio 2: 3: 5. If C's Monthly salary is ` 1,200 more than that of A, find B's monthly salary.

   (a) ₹ 2000   (b) ₹ 1000

   (c) ₹ 1500   (d) ₹ 1200

4.   Total salary of three persons A,B and C is Rs. 1,44,000. They spend 80%, 85% and 75% respectively. If their savings are in the ratio 8 : 9 : 20, find C's salary.

   (a) 48000   (b) 64000

   (c) 40000   (d) 32000

5.   The ratio of two numbers is 4:5. But, if each number is increased by 20, the ratio becomes 6:7. The sum of such numbers is:

   (a) 90   (b) 95

   (c) 100   (d) 60

6.   During the academic session 2009-10, in Banaras Hindu University, Varanasi, the number of students studying Arts, Law and Commerce was in the ratio of 5:6:7. If during the academic session 2010-11 the number of students studying Arts, Law and Commerce increased by 20%, 30% and 40% respectively, what will be new ratio?

   (a) 26:42:63   (b) 36:44:73

   (c) 26:39:49   (d) 30:39:49

7.   ₹ 324 is divided among three friends Sonu, Monu and Hari in the ratio 5:6:7. What is Monu's share of money?

   (a) 68   (b) 108

   (c) 60   (d) 120

Answers

1.   (c) Let in each month he earns = x. In March he gets = 2x. Total earning = 11x + 2x = 13x. Part earned in March = 2x/13x = 2/13.

2.   (d) In 60 litres of adulterated milk, water is = (1/3) × 60 = 20 litre. Milk = 40 litre. Let amount of water to be added be x. Now, 40/(20 + x) = 1/2 which gives x = 60 l.

3.   (d) Let monthly salary of A, B and C be 2x, 3x, 5x respectively. Now, 5x – 2x = 1200, which gives x = 400. B's salary = 3 × 400 = ₹ 1200

4.   (b) Let salary of A, B and C be x, y, z. After spending 80%, 85% and 75%, A has 20x/100, B has 15y/100 and C has 25z/100. Let their saving be 8a, 9a and 20 a. So, 20x/100 = 8a, 15y/100 = 9a and 25z/100 = 20a. Now, x + y + z = 144, 000. Putting values of x, y and z, we get a = 800. Hence, C's salary will be 80a = 80 x 800 = 64,000.

5. (a) Let the numbers are 4x and 5x. (4x + 20)/((5x + 20) = 6/7, which gives x = 10. Hence sum = 4x + 5x = 9x = 9 x 10 = 90.

6. (d) Ratio is 5: 6 : 7. After increment of 20%, 30% and 40%, new ratio will be (120/100) x 5 : (130/100) x 6 : (140/100) x 7 = 60 : 78 : 98 = 30 : 39 : 49.

7. (b) Let money received by Monu and Hari be 5x, 6x and 7x. Now, 5x + 6x + 7x = 324. This gives x = 18. Hence, Monu's share = 6 x 18 = ₹ 108.

AVERAGE

- Average = sum of observations/number of observations

- If the number of quantities in two groups are n_1 and n_2 and their average is x and y, respectively, the combined average (average of all of them put together) is

$$\frac{n_1 x + n_2 y}{n_1 + n_2}$$

- The average of n quantities is equal to x. If one of the given quantities whose value is p, is replaced by a new quantity having value q, the average becomes y, then q = p + n(y − x).

- The average of n quantities is equal to x. When a quantity is removed, the average becomes y. The value of the removed quantity is n(x − y) + y

- The average of n quantities is equal to y. When a quantity is added, the average becomes y. The value of the new quantity is n(y − x) + y.

Example

The average weight of 24 students of section A of a class is 58 Kg, whereas the average weight of 26 students of section B of the same class is 60.5 Kg. Find out average weight of all the 50 students of the class.

Answer

Here, n_1 = 24, n_2 = 26, x = 58, and y = 60.5.

∴ Average weight of all the 50 students

$$= \frac{n_1 x + n_2 y}{n_1 + n_2} = \frac{24 x 58 + 26 x 60.5}{24 + 26}$$

= 59.3 Kg.

Example

The average value of six numbers 7, 12, 17, 24, 26 and 28 is 19. If 8 is added to each number, what will be the new average?

Answer

The new average = $\bar{x} + a$ = 19 + 8 = 27.

Example

The average weight of 25 persons is increased by 2 Kg when one of them whose weight is 60 Kg, is replaced by a new person. What is the weight of the new person?

Answer

The weight of the new person = p + n(y − x) = 60 + 25(2) = 110 Kg.

Example

The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average age reduces by 1 year. What is the age of the class teacher?

Answer

The age of class teacher = n(x − y) + y = 25(16 − 15) + 15 = 40 years

Example

The mean of the marks scored by 50 students was found to be 39. Later on it was discovered that a score of 43 was misread as 23. The corrected mean is:

(a) 38.6   (b) 39.4

(c) 39.8   (d) 39.2

Answer

Wrong sum of 50 observations = 50 × wrong mean = 50 × 39 = 1,950

WE should *add* 43 - 23 = 20 in this wrong sum.

Corrected sum = 1950 + 20 = 1970

∴ Corrected mean = 1970/50 = 39.4

Example

Class XII of XYZ International School has three sections - K, L and M. In Mathematics examination, the average marks obtained by these sections are 72, 60 and 50 respectively. The average marks of sections K and L taken together is 69. If the ratio of the number of students in sections L and M is 6 : 7, what is the average score of all the three sections put together?

(a) 65   (b) 64

(c) 63.4   (d) 64.7

Answer

(d)

Example

The number of children in a camp is x and their average weight is 20 kg. If 5 children each weighing 12 kg, join the camp or if 10 children each weighing 21 kg leave the camp, the average weight in both the cases remain the same. The value of x is

(a) 18   (b) 16

(c) 15   (d) 14

Answer

(c) Total weight = 20x.

When 5 children join, average wt. becomes $(20x + 5 \times 12)/(x + 5)$ …..(1)

When 10 children leave, average wt. becomes $(20x - 21 \times 10)/(x - 10)$….. (2)

Equating (1) and (2), x = 15

Example

The average weight of three men 'X', 'Y' and 'Z' is 75 kgs. Another man 'A' joins the group and the average weight now becomes 80 kgs. If another person 'B' whose weight is 5 kgs more than 'A replaces 'X', then the average weight of 'Y', 'Z', 'A' and 'B' will be 85 kgs. What is the weight of 'X's?

(a) 84 kgs   (b) 82 kgs

(c) 78 kgs   (d) 80 kgs

Answer

Average weight of three men X, Y and Z = 75 kgs. Hence, Total weight of three men X, Y and Z = 75 × 3 = 225 kgs.

A joins the group and average weight = 80 kgs

∴ Total weight of this group = 80 × 4 = 320 kgs.

∴ Weight of A = 320 – 225 = 95 kgs.

Weight of B is 5 kg more than A.

∴weight of B = 95 + 5 = 100 kgs.

Total weight of X, Y, A and Z = 320 kgs.

Total weight of X, Y, AB and Z = 320 + 100 = 420 kgs.

Total weight of Y, Z, A and B = 85 × 4 = 340 kg.

∴ Weight of X = (420 – 340) = 80 kgs.

Example

The average weight of 14 students was calculated as 71. But it was later found that the weight of one student had been wrongly entered as 42 instead of 56 and of another as 74 instead of 32. The correct average is

(a) 75   (b) 80

(c) 68   (d) 69

Answers

Wrong total weight = 71 x 14 = 994 kg

We should add (56 - 42 = 12) and subtract (74 - 32 = 42) to get correct total weight.

Correct total weight = 994 + 12 - 42 = 964 kg

Correct average = 964/14 = 69 kg

Example

The average price of 10 pens is ₹ 12 while the average price of 8 of these pens is ₹ 11.75. Of the remaining two pens, if the price of one pen is 60% more than the price of the other, what is the price of each of these two pens?

    (a) ₹ 12, ₹ 14   (b) ₹ 5, ₹ 7.50

    (c) ₹ 8, ₹ 12   (d) ₹ 10, ₹ 16

Answer

Total Price of 10 pen = 12 × 10 = 120

Price of 8 pen = 11.75 × 8 – 94.00

Price of Remaining 2 Pen = 120 – 94 = 26

Let Price of 1st pen = x

Price of 2nd pen = 26 = (160/100)x + (260/100)x. Hence x = 10.

Example

1.   The mean of 72 items was found to be 63. If two of the items were mis-recorded as 27 and 9 instead of 72 and 90 respectively, find the correct mean.

    (a) 64.25   (b) 64.75

    (c) 63.25   (d) 65.75

2.   A library has an average of 510 visitors on Sundays and 240 on other days. What is the average number of visitors per day in the month of June beginning with a Sunday?

    (a) 250   (b) 280

    (c) 276   (d) 285

Answer

1. (b)

2. (5 x 510 + 240 x 25)/30 = 285

Example

Three maths classes: X, Y and Z take an algebra test. The average score of class X is 83. The average score of class Y is 76. The average score of class Z is 85. The average score of class X and Y is 79 and average score of class Y and Z is 81. What is the average score of classes X, Y and Z ?

    (a) 81.5   (b) 80.5

    (c) 83   (d) 78

Answer

(a) $(83x + 76y)/(x + y) = 79$ i.e. $x = 3y/4$ and $(76y + 85z)/(y + z) = 81$ i.e. $z = 5y/4$. Average $= (83x + 76y + 85z)/(x + y + z)$ Put values of x, y and z in terms of y.

Example

The average marks of a student in ten papers are 80. If the highest and the lowest score are not considered the average is 81. If his highest score is 92, what is the lowest score?

    (a) 55   (b) 60

    (c) 62   (d) 61

Answer

(b) Total marks in 10 papers = 10 × 80 = 800. Total marks in 8 papers = 8 × 81 = 648. Total marks in 2 papers = 800 – 648 = 152. Let 1st no be x. Then x + 92 = 152 i.e. x = 60

Example

1.   Of the three numbers, the first is one third of the second and twice the third. The average of these numbers is 27. The largest of these numbers is

    (a) 18   (b) 36

(c) 54   (d) 108

2.   The average monthly income of a person in a certain family of 5 members is ₹1000. What will be monthly average income of one person in the same family if the income of one person increased by ₹12,000/- per year?

(a) ₹1200/-   (b) ₹ 1600/-

(c) ₹ 2000/-   (d) ₹ 3400/-

3.   The average age of 29 boys of a class is equal to 14 years. When the age of the class teacher is included the average becomes 15 years. Find the age of the class teacher.

(a) 44 years   (b) 40 years

(c) 52 years   (d) 66 years

4.   10 sheep and 5 pigs were bought for `₹ 6,000. If the average price of a sheep is ₹ 450, find the average price of pig.

(a) ₹ 380   (b) ₹ 410

(c) ₹ 340   (d) ₹ 300

5.   The average age of three boys is 15 years. If their ages are in the ratio 3:5:7, the age of the youngest boy is

(a) 21 years   (b) 18 years

(c) 15 years   (d) 9 years

6.   Average age of ten persons learning yoga is 32 yrs. When the age of their instructor is added, the average age becomes 34 yrs. The age of their instructor is

(a) 54   (b) 52

(c) 46   (d) 56

7.   The average height of 10 students in a class is 105 cms. If 20 more students with an average height of 120 cms join them, what will be the average height be?

(a) 105 cms   (b) 110 cms

(c) 112 cms   (d) 115 cms

8.   Ms. Jhulan Goswami scores 102 runs in the 18th innings of her career and thus increases her average by 5. After the 18th inning, her average is:

(a) 17   (b) 21

(c) 26   (d) 28

9.   Before 3 years, the average age of a five member family was 17 years. A baby having been born and the average of family is now 17 years. The present age of the baby is

(a) 3 years   (b) 2 years

(b) 1 year   (d) none of these

10. The average age of A, B and C is 25 years. The Ratio of their ages is 3 : 5 : 7. Find the age of A.

(a) 21 years   (b) 18 years

(c) 15 years   (d) Data Inadequate

Answers

1.   (c) Let a, b, c are three numbers. Now a = (1/3)b and a = 2c. Average will be (a + b + c)/3 = 27. Put values of a, b and c in the terms of "a" to find the value of "a". Now proceed.

2.   (a) Salary increased per year = 12000. Hence monthly increase = 12000/12 = 1000. Average increase = 1000/5 = 200.

3.   (a) Average age of 29 boys = 14 years. When teacher included average = 15 years. Age of teacher = $30 \times 15 - 29 \times 14$ = 44 years

4.   (d) (Average price of pig x no. of pigs + average price of ship x no. of ships)/(no. of pigs + no. of ships) = total average price $\Rightarrow$ (5x + 10 x 450)/15 = 6000/15 So x = 300

5.   (d) Let age of three boys be 3x, 5x and 7x. Now (3x + 5x + 7x)/3 = 15 $\Rightarrow$ x = 3. Age of youngest boy = 3 x 3 = 9 years.

6. (a) Average age of ten person learning yoga = 32 years. Average age of 11 person including instructor = 34 years. Age of instructor $11 \times 34 - 10 \times 32 = 54$ years.

7. (d) Average height of 10 student = 105 cms. Average height of 20 student = 120 cms. Total Average height of 30 student = $(10 \times 105 + 20 \times 120)/30 = 115$ cm

8. (a) $(17x + 102)/18 = x + 5 \Rightarrow x = 12$. Increased average = $12 + 5 = 17$

9. (b) Before 3 year ago average age of 5 members = 17. Total age = $17 \times 5 = 85$. Total present age of 5 members = $85 + 15 = 100$. Present Average age of 6 members = 17. Total age = $17 \times 6 = 102$. Age of child = $102 - 100 = 2$ years

10. (c) Average age of A, B and C = 25 years Total age = $25 \times 3 = 75$. Age of A = $(3 \times 75)/(3 + 5 + 7) = 15$ years.

PERCENTAGE

- In mathematics, percentage is a way of expressing a number as a fraction of 100 (per cent meaning per hundred). It is often denoted using the percent sign, %. For example, 45% (read as "forty-five percent") is equal to $45\% = 45/100 = 0.45$

- For converting a fraction or a decimal to a Percentage, multiply it by hundred.

- For converting a percentage to a fraction or decimal, divide by hundred.

- To find a % of given number N. x% of given number (N) = $(x/100) \times N$

- If the price of a commodity increases by P%, then the reduction in consumption so as not to increase the expenditure is $\left(\dfrac{P}{100+P}\right)x100\%$

- If the price of a commodity decreases by P%, then the increase in consumption so

as not to increase the expenditure is

$$\left(\frac{P}{100-P}\right)x100\%$$

- If A is x% greater than B, then B will be

  $\dfrac{x}{(100+x)}x100\%$ lesser than A.

- If A is x% lesser than B, then B will be

  $\dfrac{x}{(100-x)}x100\%$ more than A.

- If a number is changed (increased/decreased) successively by x% and y%, then net % change is given by

  $\left(x+y+\dfrac{xy}{100}\right)$ % which represents

  increase or decrease in value according as the sign is +ve or –ve. If x or y indicates decrease in percentage, then put –ve sign before x or y, otherwise +ve sign.

- If two parameters A and B are multiplied to get a product and if A is changed (increased/ decreased) by x% and another parameter B is changed (increased/decreased) by y%, then the net % change in the product (A × B) is given

  $\left(x+y+\dfrac{xy}{100}\right)$ which represents increase

  or decrease in value according as the sign in +ve or –ve. If x or y indicates decrease in percentage, then put –ve sign before x or y, otherwise +ve sign.

- If the present population of a town (or value of an item) be P and the population (or value of item) changes at r% per annum, then

  Population (or value of item) after n years

  $$= P\left(1+\frac{r}{100}\right)^{n}$$

  Population (or value of item) n years ago

$$= \frac{P}{\left(1+\frac{r}{100}\right)^n}$$

- If a number A is increased successively by x% followed by y% and then by z%, then the final value of A will be

  $A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$ In case a given value decreases by any percentage, we will use a -ve sign before that.

- In an examination, the minimum pass percentage is x% If a student secures y marks and fails by z marks, then the maximum marks in the examination is

  $$\frac{100(y+z)}{x}$$

Example

What percentage is equivalent to 3/5?

Answer

(3/5) × 100 = 60%.

Example

What fraction is $16\frac{2}{3}$ %?

Answer

$16\frac{2}{3}\% = \frac{\left(\frac{50}{3}\right)}{100} = \frac{1}{6}$

Example

75% 0f 400 = ?

Answer

75% of 400 = (75/100) x 400 = 300

Example

What percentage of 25 kg is 3.5 kg?

Answer

Let x% of 25 kg be 3.5 kg. Then x% of 25 = 3.5. Hence (x/100) x 25 = 3.5 $\Rightarrow$ x = 14%

Example

If Mohan's salary is 10% more than that of Sohan, then how much per cent is Sohan's salary less than that of Mohan?

Answer

Here, x = 10. Required answer = will be

$\left(\frac{x}{100+x}x100\right)\% = \left(\frac{10}{100+10}x100\right)\% = 9\frac{1}{11}\%$

Example

If A is 20% of C and B is 25% of C, then what percentage is A of B?

Answer

Here, x = 20 and y = 25.

$A = \frac{x}{y}x100\% \, of \, B = \frac{20}{25}x100 = 80\% \, of \, B$

Example

If the price of sugar increases by 25%, find how much per cent its consumption be reduced so as not to increase the expenditure.

Answer

Reduction in consumption

$= \left(\frac{P}{100+P}x100\right)\% = \left(\frac{25}{100+25}x100\right) = 20\%$

Example

If salary of a person is first increased by 15% and thereafter decreased by 12%, what is the net change in his salary?

Answer

Here, x = 15 and y = –12.

The net% change will be

$$= \left(x + y + \frac{xy}{100} \right)\% = \left(15 - 12 - \frac{15x12}{100} \right) = 1.2\%$$

Example

If the side of a square is increased by 20%, its area is increased by k%. Find the value of k.

Answer

Since side × side = area

∴ Net% change in area

$$= \left(x + y + \frac{xy}{100} \right)\% = \left(20 + 20 + \frac{20x20}{100} \right) = 44\%$$

Example

The population of a town increases 5% annually. If its present population is 84000, what will it be in 2 years time?

Answer

Here, P = 84000, r = 5 and n = 2.

∴ Population of the town after 2 years

$$= P\left(1 + \frac{r}{100} \right)^n = 84000\left(1 + \frac{5}{100} \right)^2 = 92610$$

Example

The population of a town is 144000. It increases by 5% during the first year. During the second year, it decreases by 10% and increases by 15% during the third year. What is the population after 3 years?

Answer

Here, P = 144000, x = 5, y = – 10 and z = 15.

∴ Population of the town after 3 years

$$= A\left(1 + \frac{x}{100} \right)\left(1 + \frac{y}{100} \right)\left(1 + \frac{z}{100} \right)$$

$$= 144000\left(1 + \frac{5}{100} \right)\left(1 - \frac{10}{100} \right)\left(1 + \frac{15}{100} \right) = 156492$$

Example

The Municipality of a town increases water tax by 20% and water consumption decreased by 20%. Then the percentage of increase or decrease in the monthly expenditure is:

    (a) 4% increase

    (b) 4% decrease

    (c) 5% increase

    (d) 5% decrease

Answer

(b)

Example

a and b are inversely proportional to each other and are positive. If a increases by 100%, then b decreases by

    (a) 50%   (b) 75%

    (c) 100%   (d) 200%

Answer

(a) a and b are inversely proportional, Hence we can say a = k/b or ab = k or b = k/a. Here k is constant. When "a" increases by 100% then it becomes a + a(100/100) = 2a. Now we can say that b' = k/2a. Hence decrease in b =

$$\left(\frac{b - b'}{b} \right)x100 = \frac{\left(\dfrac{k}{a} - \dfrac{k}{2a} \right)}{k/a}x100 = 50\%$$

Example

2.   The length of a square is increased by 15% and breadth decreased by 15%. The area of the rectangle so formed is

    (a) neither increases nor decreases

    (b) decreases by 2.25%

    (c) increases by 2.25%

    (d) decreases by 22.5%

3.   The petrol prices shot up by 7% as a result of the hike in the price of crudes.

The price of petrol before the hike was ₹ 28 per litre. Vawal travels 2400 kilometers every month and his car gives a mileage of 18 kilometers to a litre. Find the increase in the expenditure that Vawal has to incur due to the increase in the price of petrol (to the nearest rupee)?

(a) ₹ 270

(b) ₹ 262

(c) ₹ 276

(d) ₹ 272

4. Two cubes of bronze have their total weight equivalent to 60 kg. The first piece contains 10 kg of pure zinc and the second piece contains 8 kg of pure zinc. What is the percentage of zinc in the first piece of bronze if the second piece contains 15 per cent more zinc than the first?

(a) 15%

(b) 25%

(c) 55%

(d) 24%

6. Ram's income is 20% less than Shyam's. How much is Shyam's income more than Ram's in percentage terms?

(a) 20%

(b) 25%

(c) 30%

(d) 15%

7. The population of a town is 155625. For ever 1000 males there are 1075 females. If 40% of the males and 24% of the females are literate, find the percentage of literacy in the town.

(a) 33.7

(b) 32.7

(c) 31.7

(d) 30.7

10. Which one of the following is same as 30% of 40% of 560?

(a) 60% of 40% of 280

(b) 15% of 80% of 280

(c) 30% of 40% of 280

(d) 15% of 80% of 140

15. The price of an article is cut by 10%. To restore it to the original value, the new price must be increased by

(a) 10%

(b) $9\dfrac{1}{11}\%$

(c) 11%

(d) $11\dfrac{1}{9}\%$

16. If 0.06% of a number is 84, then 30% of that number is

(a) 25.2

(b) 420

(c) 42000

(d) 2520

18. In an examination, a student who secured 25% of the maximum marks fails by 60 marks but another candidate who secures 45% of the maximum marks gets 10 marks more than required passing marks. The maximum number of marks is

(a) 450

(b) 350

(c) 525

(d) none of these

Answer

2. (b) Decrease will be

$$\left(x - y - \frac{xy}{100}\right) = 15 - 15 - \frac{15x15}{100} = 2.25\%$$

3. Petrol consumption = 2400/18; total cost of petrol = 28 x (2400/18). Hence, increase after hike = (7/100) x 28 x (2400/18) = ₹ 262

4. (b) Let first and second piece of bronze are x and (60 - x) kgs.

$$\frac{8}{60-x} - \frac{10}{x} = \frac{15}{100} \Rightarrow x = 40\% = \frac{10}{40}x100 = 25\%$$

6. (b) Let Shyam income = 100; Ram's income = 80. Shyam's income is more by

$$= \frac{100-80}{80}x100 = 25\%$$

7. (c) Ratio of male to female = 1000:1075 = 40 : 43; 40% of male and 24% of female are literate i.e. total % of literacy

$$= \frac{\dfrac{40}{100}x40 + \dfrac{24}{100}x43}{40+43}x100 = 31.7\%$$

10. (a) 30% of 40% of 560 = (30/100) x (40/100) x 560 = 67.2. Which is equal to

60% of 40% of 280 = (60/100) x (40/100) x 280 = 67.2.

15. (d) Let the article of 100 be cut to 90 by 10%. Then to restore to 100, Percentage will be (10/90) x 100 = $11\frac{1}{9}$%

16. (c) Let the number be x. Now 0.06x/100 = 4. Hence x = (84/0.06) x 100. Now, 30% of x = (30/100) x (84/0.06) x 100 = 42000

18. (b) By getting 25% marks one fails by 60 marks. By getting 45% marks other get 10 marks more than required passing marks. Let Maximum marks = x.

(45 - 25)% of x = 70 ⇒(20/100) x = 70 i.e. x = 350.

SIMPLE INTEREST & COMPOUND INTEREST

Simple Interest

- Simple interest (SI) = PRT/100

  Here P is principal amount, R is rate per annum and T is time in years.

- Amount (A) will be

  $$= P + \frac{PRT}{100} = P\left(1 + \frac{RT}{100}\right) = P + SI$$

Compound Interest

- If interest is compounded annually

  Amount = A = $P\left(1 + \frac{R}{100}\right)^N$

- If interest is compounded half yearly

  Amount = A = $P\left(1 + \frac{R}{200}\right)^{2N}$

- If interest is compounded quarterly

  Amount = A = $P\left(1 + \frac{R}{400}\right)^{4N}$

- If rate of interest changes over the years

Amount A = $P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)$.......

- Compound interest = Amount - Principal

- Difference between CI and SI

  for two years = $P\left(\frac{R}{100}\right)^2$

  for three years = $\frac{PR^2(300 + R)}{(100)^3}$

Example

Find the simple interest and amount when ₹ 1000 is lent at 5% per annum for 5 years.

Solution

$$SI = \frac{PRT}{100} = \frac{1000 x 3 x 2}{100} = 60$$

∴ Amount A = P + SI = 100 + 60 = ₹ 160

Example

In how many years will the sum of ₹ 500 become ₹ 620 if the rate of simple interest is 4% per annum?

Answer

$$T = \frac{SI x 100}{R x P} = \frac{(620 - 500) x 100}{500 x 4} = 6$$

∴ Time = 6 years

Example

Find the compound interest on ₹ 2000 at 5% per annum for 3 years, compound annually.

Answer

Here P = ₹ 2000, R = 5

$$CI = 2000\left[\left(1 + \frac{5}{100}\right)^3 - 1\right] = ₹ 315.25$$

Example

The difference between compound interest and simple interest on a certain sum of money in 3 years at the rate of 7% per annum is ₹ 225.645. What is the principal?

Answer

Difference of interest = $P\left(\dfrac{R^2(300+R)}{(100)^3}\right)$

$\therefore\ 225.645 = P\left(\dfrac{7^2(300+7)}{100^3}\right)$

$\therefore$ P = ₹ 15000

Example

Mohan borrows ₹ 10,000 from two money lenders at a rate of 6% pa and 8% pa respectively, for a period of 3 years. If the total interest he paid was ₹ 1980, find the amount borrowed at the rate of 6% pa.

Answer

Let the amount borrowed at 6% is x and amount borrowed at 8% is y.

$\qquad$ x + y = 10,000 $\qquad$ (1)

Sis will be 18x/100 and 24y/100.

$\therefore$ From given condition

$\qquad \dfrac{18x}{100} + \dfrac{24y}{100} = 1980 \qquad$ (2)

Solving (1) and (2); x = 7000, y = 3000

$\therefore$ Amount borrowed @ 6% = ₹ 7000

Example

1. An employee of an organization invests a total of Rs 25,400 in two different schemes X and Y at a simple interest rate of 18% per annum and 10% per annum respectively. If a total of Rs. 6460 has been earned as simple interest in 2 years, what amount was invested in Scheme Y?

(a) Rs. 8,625 $\qquad$ (b) Rs. 16,775

(c) Rs. 12,240 $\qquad$ (d) Rs. 10,930

2. The difference between Simple Interest and Compound Interest on Rs. 500 for 1 year at 10% per annum, reckoned half yearly is

(a) Rs. 1 $\qquad$ (b) Rs. 1.25

(c) Rs. 1.5 $\qquad$ (d) Rs. 2

Answer

1. (b) $\qquad$ 2. (b)

Example

A sum amounts to ₹ 9680 in 2 years and to ₹ 10648 in 3 years respectively at compound interest. What will be the amount if the same sum is invested for $1\dfrac{2}{5}$ years at the same rate of compound interest?

(a) 9025 $\qquad$ (b) ₹ 9152

(c) ₹ 9215 $\qquad$ (d) ₹ 9251

Answer

(b) $\quad 9680 = P\left(1+\dfrac{R}{100}\right)^2 \qquad$ (1)

$\qquad 10648 = P\left(1+\dfrac{R}{100}\right)^3 \qquad$ (2)

Solving R = 10%. From (1) P = ₹ 8000

Now t = $1\dfrac{2}{5}$ years = 1.4 years

Required amount A = $8000\left(1+\dfrac{10}{100}\right)^{1.4}$ = ₹ 9152.

Example

A certain sum of money amounts to ₹ 1,008 in 2 years and to ₹ 1,164 in $3\dfrac{1}{2}$ years. Find the sum and the rate of interest.

(a) ₹ 900 and 12%

(b) ₹ 800 and 12%

(c) ₹ 700 and 13%

(d) ₹ 800 and 13%

Answer

(d) ₹ 1008 in 2 years; ₹ 1164 in $3\dfrac{1}{2}$ years

SI on 1008 for $1\dfrac{1}{2}$ year = 1164 – 1008 = 156

SI for 1 year = (156/3) x 2 = 104

SI for 2 years = 208

Principal + SI = 1008

Principal in starting = 1008 – 208 = 800

Rate of interest =

(104/800) x 100 = 13%

Example

The difference between simple interest and compound interest at the same rate for rupees 5,000 for two years is rupees 98. The rate of interest is:

(a) 14%   (b) 10%

(c) 10.5%   (d) 12%

Answer

(a) P = 5000, T = 2 years, CI - SI = 98

Now   CI - SI = $\dfrac{PR^2}{(100)^2}$

$\therefore$ $98 = \dfrac{5000R^2}{(100)^2} \Rightarrow R = 14\%$

Example

What will be the difference in simple and compound interest on ₹ 2, 000 after three years at the rate of 10 percent per annum?

(a) ₹ 60   (b) ₹ 42

(c) ₹ 62   (d) ₹ 40

Answer

(c) CI - SI = $P\left(1+\dfrac{R}{100}\right)^3 - P - \dfrac{PRT}{100}$

Put P = 2000, R = 10, T = 3

We get Difference = ₹ 62

Example

1. A man buys ₹ 20 shares paying 9% dividend. The man expects to have an interest of 12% on his money. The market value of each share is:

    (a) ₹ 12   (b) ₹ 15

    (c) ₹ 21   (d) ₹ 18

2. The value of a machine depreciates every year at the rate of 10% on its value at the beginning of that year. If the present value of the machine is ₹ 729, its worth three years ago was:

    (a) ₹ 1,000   (b) ₹ 750.87

    (c) ₹ 800   (d) ₹ 947.70

Answer

1. (b) Let the market value of share be x. Now, 12% of x = 9% of 20. Hence x = 15.

2. (a) Let value of Machine 3 years ago = x. Depreciation rate = 10% per annum. Present value = 729.

    $x\left(1-\dfrac{10}{100}\right)^3 = 729 \Rightarrow$ x = ₹ 1000.

Example

1. A sum is divided between A and B in the ratio of 1 : 2. A purchased a car from his part, which depreciates $14\dfrac{2}{7}$ per annum and B deposited his amount in a bank, which pays him 20% interest per annum compounded annually. By what percentage will the total sum of money

increase after two years due to this investment pattern (approximately).

(a) 20%   (b) 26.66%

(c) 30%   (d) 25%

2.   Ram borrows Rupees 520 from Govind at a simple interest of 13% per annum. What amount of money should Ram pay to Govind after six months to be absolved of the debt?

(a) 552.80   (b) 553.80

(c) 453.80   (d) 353.80

3.   Kirti took loan of ₹ 800 at the rate of 11% per year for 7 months. After seven months she has to pay

(a) ₹ 851   (b) ₹ 852

(c) ₹ 950   (d) ₹ 951

4.   The number of the members of a club is increased by 10% every year. If the initial number of the members is 500, then what will be the number at the beginning of the third year?

(a) 610   (b) 615

(c) 620   (d) 605

5.   The value of a car depreciates every year at the rate of 10% on its value at the beginning of the year. If the present value of the car is ₹ 52,488, its worth four years ago was

(a) ₹ 68.232   (b) ₹ 68.234

(c) ₹ 69.862   (d) ₹ 80,000

6.   In a factory, the production of scooters rose to 48400 from 40000 in 2 years. The rate of growth per annum is

(a) 20%   (b) 10%

(c) 30%   (d) 8%

7.   A certain sum of money was deposited in bank and it became two-fold in 10 years, what is the rate of simple interest?

(a) 8%   (b) 10%

(c) 12%   (d) 13%

8.   If ₹ 80 amounts to ₹ 140 in 4 years, what will ₹ 96 amount to in 10 years at the same rate of interest per annum?

(a) ₹ 276   (b) ₹ 306

(c) ₹ 386   (d) ₹ 300

9.   The difference between the simple interest and the compound interest (compounded annually) on ₹ 2000 for 2 yr at 8% per annum will be

(a) ₹ 10   (b) ₹ 20

(c) ₹ 13   (d) ₹ 25

Answers

1.   (a) Depreciation $14\frac{2}{7}$ % per annum means 100/7 % per annum.

Case 1: $A = x\left(1 - \frac{100/7}{100}\right)^2 = \frac{36x}{49}$

Case 2: $A = 2x\left(1 + \frac{20}{100}\right)^2 = \frac{72x}{25}$

Total A $= \frac{36x}{49} + \frac{72x}{25}$

Increase $= \frac{36x}{49} + \frac{72x}{25} - 3x = \frac{753x}{49x25}$

$\therefore$ % increase $= \frac{753x}{49x25x3x} x100 = 20\%$ approx

2.   (b) SI = PRT/100;

P = 520, T = 1/2, R = 13%

$\therefore$ SI = 520 x 1 x 13/2 x 100 = 33.80

$\therefore$ A = 520 + 33.80 = 553.80

3.   (a) P = ₹ 800, R = 11%, T = 7

$\therefore$ SI = PRT/100 = 800 x 11 x 7/100 x 12 = 51. Hence A = 800 + 51 = ₹ 851

4.   (d) P = 500, R = 10, T = 2

$$A = 500\left(1+\frac{10}{100}\right)^2 = 605$$

5.   (d) A = 52,488, R = 10, T = 4, Find P.

6.   (b) P = 40,000, A = 48,400, T = 2 Find R

     using $A = P\left(1+\frac{R}{100}\right)^T$

7.   (b) Let Principal = P, Amount = 2P, SI = 2P – P = P, R = ?, T = 10 year. Find R.

8.   (a) Case 1: P = 80, A = 140, SI = 140 - 80 = 60, R = ?, T = 4. Find R which comes out (75/4)%

     Case 2: P = 96, R = 75/4, T = 10, SI = ?, A = ? From which SI = 180 and A = 276 (do it yourself).

9.   (c) Diff in CI and SI for two years

     $$= P\left(\frac{R}{100}\right)^2 = 2000\left(\frac{8}{100}\right)^2 = 12.8$$

PROFIT, LOSS AND DISCOUNT

- **Cost Price**: The cost price of an article is the price at which an article has been purchased. It is abbreviated as C.P.

- **Selling Price**: The selling price of an article is the price at which an article has been sold. It is abbreviated as S.P.

- **Profit or Gain**: If the selling price of an article is more than the cost price, there is a gain or profit. Thus, Profit or Gain = S.P. - C.P.

- **Loss:** If the cost price of an article is greater than the selling price, the seller suffers a loss. Thus, Loss = C.P. - S.P.

  ☞ Note that profit and loss are always calculated with respect to the *cost price* of the item.

- Profit (or gain) = selling price - cost price

- Loss = cost price - selling price

- Profit % = $\dfrac{profit \, x \, 100}{cost \, price}$

- Loss % = $\dfrac{loss \, x \, 100}{cost \, price}$

- Selling price = $\dfrac{100 + Gain\%}{100} x \, cost \, price$

- Selling price = $\dfrac{100 - Loss\%}{100} x \, cost \, price$

- Cost price = $\dfrac{100}{100 + Gain\%} x \, selling \, price$

- Cost price = $\dfrac{100}{100 - Loss\%} x \, selling \, price$

- A *dishonest* shopkeeper claims to sell goods at cost price, but uses a lighter weight, then his Gain %

  $= \dfrac{100 \, x \, excess}{original \, value - excess}$

- If two items are sold each at rupees R, one at a gain of x % and other at a loss of x %, there is always an overall loss given by $\dfrac{x^2}{100}\%$ and the value of loss is given by $\dfrac{2x^2 R}{(100^2 - x^2)}$.

- ☞ If an article is sold at a gain of say, 20% then S.P = 120% of C.P. So, instead of first finding 20% of CP and then adding, it would be simple to calculate it as given above. Also, if an article is sold at a loss of say, 20%, then S.P = 80% of CP.

- Discount = Marked price - selling price

  Marked price (MP) is also called List price (LP). This price is fixed by the shopkeeper over and above the selling price in anticipation that he would be asked for a discount. It is also known as market- up price.

MATHEMATICS
MATHEMATICS I

$$LP \xrightarrow{\%decrease} SP \xrightarrow{\%Profit/Loss} CP$$

- Rate of discount = $\dfrac{Discount}{Marked\ price} x100$

- Markup% = $\left(\dfrac{MP}{CP}-1\right)x100$

- Computation of discount percent when buy 'x' get 'y' free scheme is launched:

  Discount% = $\dfrac{Free\ units}{Total\ units}x100$

Example

If the cost price of a book is ₹ 150 and selling price is 137.50, then calculate the loss and percentage loss on the book?

Answer

Cost price = ₹ 150 and selling price = ₹ 137.50

∴ Loss = Cost price – selling price = (150 – 137.50) = ₹ 12.50

Loss % = $\dfrac{loss\ x100}{cost\ price} = \dfrac{12.50 x100}{150} 8.33\%$

Example

A shopkeeper professes to sell sugar at cost price, but uses a false weight which reads 1000 gms for 900 gm. What is his profit percent?

Answer

Gain% = $\dfrac{100\ x\ excess}{original\ value - excess}$

Here excess = 1000 - 900 = 100 gm

∴ Gain% = $\dfrac{100(100)}{1000-100}=11.11\%$

Example

Ram sells two Mobile phones for ₹ 1000 each, one at a profit of 10% and other at a loss of 10%. Find his gain or loss percentage.

Answer

Loss % = $\left(\dfrac{x^2}{100}\right)\% = \dfrac{10 x10}{100}=1\%$

Loss = $\dfrac{2x^2 R}{(100^2 - x^2)} = \dfrac{2 x10^2 x1000}{100^2 - 10^2} = $ ₹ 20.20

Example

Big Bazaar is offering "Buy 2, get 1 free" on household items. What is the net percentage discount being offered by the store?

Answer

Discount% = $\dfrac{Free\ units}{Total\ units}x100$

∴   D%= $\dfrac{1}{(2+1)}x100 = 33.33\%$

Example

After selling an article at a discount of 20%, profit percentage obtained is 10%. What is the mark-up over CP?

Answer

0.8 MP = 1.1 CP

MP/CP = 1.1/0.8

Markup% =

$\left(\dfrac{MP}{CP}-1\right)x100 = \left(\dfrac{1.1}{0.8}-1\right)x100 = 37.5\%$

Example

A man purchased two watches for 560. He sold one at a 15% profit and the other at a 10% loss, and thus he neither gains nor loses. Find the cost price of each watch.

    (a) 320, 240   (b) 240, 320

    (c) 224, 336   (d) 336, 224

Answer

(c)   CP1 × 0.15 = CP2 × 0.10

CP1/CP2 = 2/3   (i)

CP1 + CP2 = 560   (ii)

On comparing (i) and (ii)

CP1 = 224 and CP2 = 336

Example

A radio costing `₹ 500 is available on 10% discount on cash purchase. The shopkeeper gives sequential discounts. Asha paid ₹ 427.50 for the radio. Find the rate of the discount on cash price.

(a) 20%   (b) 15%

(c) 10%   (d) 5%

Answer

Price after 1st discount of 10% = 90% × 500

= 450

If final cash price is ₹ 427.50

Discount % = (22.5/450) x 100 = 5%

Example

Naresh bought a bicycle each for his two sons, each bicycle priced at Rs. 3500. If the first bicycle is sold at a profit of 5%, the how much should the other bicycle be sold for, to gain a total of 20% on both?

(a) 15%   (B) 10%

(c) 25%   (d) 35%

Answer

(d)

Example

Amrit wants to sell cars at 25% above the cost price. Due to a slump in the market, his cost reduces by 5%. He thus offers a discount of 8% due to which the sales go up by 25%. Compute the change in Amrit's profit.

(a) unchanged   (b) 5%

(c) 4%   (d) 3%

Answer

(a)

Example

1. The marked price of an article is 20% more than the cost price. If the article is sold at a discount of 15% on its marked price, then the gain percent is

(a) 5   (b) $4\dfrac{1}{2}$

(c) $2\dfrac{1}{2}$ (c) 2

2. Reshma sells an article to Rekha at 37.5% profit, Rekha sells it to Madhu at $9\dfrac{1}{11}$ % profit. Again Madhu sells it to Mitu at 25% loss. If Mitu pays ₹342 for the article, then what is the cost price of the article to Reshma?

(a) ₹ 304   (b) ₹ 266.50

(c) ₹ 380   (d) ₹ 384.75

Answer

1. Let cost price is ₹ 100. Market price will be 100 x (120/100) = ₹ 120. Selling price will be 120 x (85/100) = ₹ 102. Profit will be [(102 - 100)/100] x 100 = 2%.

2. Cost price of the article to Reshma

$$= 342x\frac{100}{(100-25)}x\frac{100}{(100+9.09)}x\frac{100}{(100+37.5)}$$

= ₹ 304.

Example

1. A trader sells rice at a profit of 20% and uses weights which are 10% less than the correct weight. The total gain earned by him is:

(a) $22\dfrac{2}{9}$ (b) 35%

(c) $33\frac{1}{3}$ (d) 30%

2. A shirt was sold at a profit of 15%, If its cost had been 5% less and it had been sold for ₹ 21 less, then the profit would have been 10%. Find the cost of the shirt.

(a) ₹ 210   (b) ₹ 220

(c) ₹ 100   (d) ₹ 200

Answer

1. (c) Let us consider of a packet of rice marked 1 kg. Its actual weight is 90% of 1000 gm = 900 gm. Let C.P. of each gm = ₹1. Then, C.P. of this packet = ₹ 900. S.P. of this packet = 120% of C.P. of 1 kg = (120/100) x 1000 = ₹ 1200. Gain% = $\left(\frac{300}{900}x100\right)\% = 33\frac{1}{3}\%$

2. (d) Let the cost price = x ; selling price = 115% (x). New cost price = 95%(x) ; New selling price = 115% (x) – 21. But new SP = 110% of new CP = 110% (95% x). Therefore, 110% (95% x) = 115% (x) – 21 $\Rightarrow$ x = 200.

Example

A shop gives 15% discount on the purchase of a T.V. If paid for in cash immediately, a further discount of 12% is given. If the marked price is ₹ 15,000, what is the price of the T.V if cash purchase is made?

Answer

(b) Price of T.V. after successive discounts of 15% and 12% will be

$$15000x\left(1-\frac{15}{100}\right)\left(1-\frac{12}{100}\right) = ₹\ 11220$$

Example

A and B started a business with a total capital of ₹ 30,000. At the end of the year, they shared the profit in the ratio of their

investments. If their capitals were interchanged, then A would have received 175% more than what he actually received. Find out the capital of B.

(a) ₹ 20,000   (b) ₹ 22,000

(c) ₹ 21,000   (d) ₹ 23,000

Answer

(b) Let A contributed x and B contributed (30,000 - x). Then they will get profit in ratio $\left(\frac{x}{30000-x}\right)$. When ration is exchanged then $\left(\frac{30000-x}{x}\right)$.

Now $(30000-x)-x = \frac{175}{100}x$

$\Rightarrow$ x = 8000

$\therefore$ 30,000 - x = 30,000 - 8000 = ₹ 22000

Example

17. A dealer marked his goods 20% above the cost price and allows a discount of 10%. Then, the gain percent is

(a) 2%   (b) 4%

(c) 6%   (d) 8%

Answer

(d) Let CP be = 100

MP 20% above CP = 120

After allowing Discount of 10%

SP = (90/100) x 120 = 108

Gain = 8%

Example

1. A single discount equivalent to a discount series 15% and 5% is

(a) 19.25%   (b) 20%

(c) 10%   (d) 8.5%

2. By selling a cycle for ₹ 2345/-, a student loses 19%. His cost price is nearly

   (a) ₹ 4,000/-   (b) ₹ 5,000/-

   (c) ₹ 3,000/-   (d) ₹ 3,500/-

3. If selling price of 10 articles is equal to cost price of 11 articles, then gain is

   (a) 8%   (b) 9%

   (c) 8.5%   (d) 10%

Answer

1. (a) 15 + 5 - (15 x 5)/100 = 19.25

2. (c) SP = 2345; L = 19%; CP = (100/81) x 2345 = 3000

3. (d) Let CP of 1 article = ₹ 1. CP of 11 article = ₹ 11. CP of 10 article = ₹ 10. SP of 10 article = ₹ 11. Hence, Profit% = (11–10)/10 × 100 = 10%

Example

1. P sells a table to Q at a profit of 10% and Q sells it to R at a profit of 12%. If R pays ₹ 246.40 for it, then how much had P paid for it?

   (a) 200.00   (b) 300.00

   (c) 248.00   (d) 346.00

2. A dealer buys an article for `380.00. What price should he mark so that after allowing a discount of 5% he still makes a profit of 25% on the article?

   (a) ₹ 500   (b) ₹ 475

   (c) ₹ 95   (d) ₹ 465

Answer

1. (a) Let P paid for table = ₹ P. After all R paid = ₹ 246.40. According to the question,

$$246.40 = P\left(\frac{110}{100}\right)\left(\frac{112}{100}\right) \Rightarrow P = 200$$

2. (a) CP = 380. SP = (125/100) × 380

Let MP = x. According to the question,

$$\frac{95x}{100} = \frac{125}{100}x380 \Rightarrow x = 500$$